D-brane and Gauge Invariance
in Closed String Field Theory

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Abstract

We construct a system of bosonic closed string field theory coupled to a D-brane. The interaction between the D-brane and closed string field is introduced using the boundary state which is a function of constant field strength $F_{\mu\nu}$ and the tilt θ_μ^i of the D-brane. We find that the gauge invariance requirement on the system determines the $(F_{\mu\nu}, \theta_\mu^i)$-dependence of the normalization factor of the boundary state as well as the form of the purely $(F_{\mu\nu}, \theta_\mu^i)$ term of the action. Correspondence between the action in the present formalism and the low energy effective action (bulk + D-brane actions) in the σ-model approach is studied.

1 Introduction

D-branes [1, 2, 3], "solitonic" objects carrying Ramond-Ramond charges, play crucial roles in non-perturbative understanding of string theories based on dualities [4]. Their existence was expected from various dualities including $SL(2, \mathbf{Z})$ duality in type IIB closed superstring theory [5], and the corresponding classical solutions with Ramond-Ramond charges were found in supergravity theory which describes the low energy dynamics of superstring theory [6]. Then a microscopic construction of such "solitonic" objects was given as hypersurfaces on which Dirichlet open strings end [1]. Though first introduced as fixed hypersurfaces, D-branes become dynamical objects with internal degrees of freedom originating from Dirichlet open strings. For example, a part of the massless gauge fields ("photons") are converted into the degrees of freedom representing the collective coordinates for translation of D-branes.

The purpose of this paper is to introduce D-branes in covariant bosonic string field theory (SFT), a formulation of string theory as a straightforward extension of gauge field theories. The most ambitious and interesting approach to D-branes using SFT would be identify D-branes as classical solutions (solitons) in closed SFT and carry out the quantization around D-branes using the technique familiar in local field theories [7]. This attempt, however, seems very hard and might be impossible in view of the $1/g$ dependence of the D-brane tension on the closed string coupling constant g. Instead, we shall adopt another way of introducing D-branes into closed SFT. This is to add to the SFT action a term describing the interaction between D-brane and closed string. This interaction will be given as a product $B \cdot \Phi$ of closed string field Φ and the "boundary state" B [8, 9, 10, 11, 12]. The latter has been known to describe the initial (final) state of a closed string emitted from (absorbed by) the D-brane.

Of course, we need a principle for introducing such a new interaction. Our principle here is to keep the stringy local gauge invariance present in the original closed SFT. This stringy invariance includes, for example, the general coordinate invariance and the gauge invariance associated with massless anti-symmetric tensor field. In order to preserve the gauge invariance after introducing the $B \cdot \Phi$ interaction, the boundary state B must also transform. We shall see that this is realized by defining the transformation law of the dynamical variables W associated with the boundary state. The gauge invariance requirement also fixes the W-dependence of the normalization factor of the boundary state as well as the purely W-term which we have to add to the SFT action besides the $B \cdot \Phi$ interaction. In this paper we take as W, the dynamical variable associated with a D-brane, constant field strength $F_{\mu\nu}$ and the parameter

θ^i_μ representing the tilt of the D-brane.

Unfortunately, we do not yet have a fully satisfactory quantum theory of covariant closed SFT. The origin of the problem is that, although we have closed SFT actions having gauge invariance [13, 14, 15], their naive path-integral quantization leads to theories where gauge invariance and unitarity are broken. One way to remedy this defect is to add quantum corrections to classical SFT action [16, 17] by following the Batalin-Vilkovisky formalism [18]. However, the resulting theories become too complicated to be used for practical analysis. In this paper, we ignore this quantization problem and construct a "closed SFT + D-brane" system on the basis of covariant closed SFT proposed in refs. [13, 14]. We hope that the essentials of this paper remain valid in a more complete formulation.

Finally, we mention another way of describing D-branes in the framework of SFT which we do not adopt in this paper. This is to consider a field theory of Dirichlet open string. Given a SFT for Neumann open string (e.g., the ones given in [19, 20]), the Dirichlet open SFT is obtained by T-duality transformation [21, 22]: in the BRST charge and the string vertices we have only to replace the center-of-mass momentum in the transverse directions with the difference between the coordinates of the two D-branes on which the open string end. However, in this approach the closed string degrees of freedom are treated rather indirectly since they appear dynamically as loop effects in covariant open SFT.

The organization of the rest of this paper is as follows. In Sec. 2, we construct "closed SFT + D-brane" system using the gauge invariance principle mentioned above. The gauge transformation considered is the one which shifts the anti-symmetric tensor field in Φ by a constant. In Sec. 3, we examine the gauge invariance under linear coordinate transformation. Though in Secs. 2 and 3 we take only $F_{\mu\nu}$ as dynamical variable associated with D-brane, in Sec. 4 we introduce the variable θ^i_μ specifying the tilt of the D-brane. In Sec. 5, the correspondence between the σ-model approach and the present SFT approach is studied. The final section is devoted to a summary and discussions. In Appendix A, we summarize various formulas in SFT used in the text, and in Appendix B, we present the details of the calculation of the star products used in Secs. 2, 3 and 4.

2 Introducing D-brane to SFT

2.1 Source term and gauge invariance

We start with the system of closed SFT field Φ described by the action [14],

$$S_0[\Phi] = \frac{1}{g^2} \left\{ \frac{1}{2} \, \Phi \cdot Q_{\mathrm{B}}\Phi + \frac{1}{3} \Phi \cdot (\Phi * \Phi) \right\}, \tag{2.1}$$

which has an invariance under the stringy local gauge transformation δ_Λ,

$$\delta_\Lambda \Phi = Q_B \Lambda + 2\Phi * \Lambda. \tag{2.2}$$

In eqs. (2.1) and (2.2) the meaning of the products $\cdot$ and $*$ are as given in ref. [14]. Compared with the closed string field Φ in ref. [14], we have rescaled it by the coupling constant g for later convenience.

We would like to extend this closed string field system to the one containing the D-brane degrees of freedom. As explained in Sec. 1, our principle of the extension is to keep the gauge invariance (2.2) intact. Since D-brane can be regarded as a source of closed strings, let us add to (2.1) the following source term

$$S_{\mathrm{source}} = B[W] \cdot \Phi = \int dz_0 \, \langle B[W] | \Phi \rangle \,. \tag{2.3}$$

Here, $|B[W]\rangle$ represents the state for the emission and absorption of closed strings. It is a function of new dynamical degrees of freedom associated with the D-brane, which we denote collectively by W. The integration measure dz_0 is over the zero-modes of the string coordinates $Z(\sigma) \equiv \left(X^M(\sigma), c(\sigma), \overline{c}(\sigma) \right)$ and the string-length parameter. In the following we adopt the π_c^0-omitted formulation [14] and the representation $z_0 \equiv (x^M, \overline{c}_0, \widetilde{\alpha})$, where $\widetilde{\alpha}$ is the variable conjugate to the string-length parameter α.[*] Since the string field Φ and the measure dz_0 have ghost number $N_{\mathrm{gh}}[\Phi] = -1$ and $N_{\mathrm{gh}}[dz_0] = 1$, respectively, $B[W]$ should carry $N_{\mathrm{gh}}[B] = 0$. The Grassmann integration over $\overline{c}_0$ is defined by $\int d\overline{c}_0 \, \overline{c}_0 = 1$.

Assuming that the transformation law (2.2) of the closed string field Φ remains unchanged after introducing S_{source}, the requirement that the system described by the action $S_0 + S_{\mathrm{source}}$ be invariant under the gauge transformation implies that the equation,

$$0 = \delta_\Lambda \left(S_0 + S_{\mathrm{source}} \right) = \delta_\Lambda S_{\mathrm{source}} = \delta_\Lambda B \cdot \Phi + B \cdot \left(Q_{\mathrm{B}}\Lambda + 2\Phi * \Lambda \right), \tag{2.4}$$

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[*] String field Φ in SFT of refs. [14, 20] contains as its argument the string-length parameter α in addition to $\left(X^M(\sigma), c(\sigma), \overline{c}(\sigma) \right)$. The $\widetilde{\alpha}$-representation is obtained from the α-representation by the Fourier transformation $\int d\alpha \exp\left(i\alpha\widetilde{\alpha} \right)$: α is a momentum-like variable while $\widetilde{\alpha}$ is a coordinate-like one. Physical quantities do not depend on $\widetilde{\alpha}$, and we have an infinite number of equivalent worlds specified by $\widetilde{\alpha}$.

holds for any Φ and any Λ. This leads to the following two conditions:

$$Q_{\mathrm{B}} B[W] = 0, \tag{2.5}$$

$$\delta_\Lambda B[W] = 2B[W] * \Lambda. \tag{2.6}$$

Namely, $B[W]$ must be a BRST invariant state annihilated by Q_{B}, and the gauge transformation law $\delta_\Lambda W$ must be determined so as to satisfy the second condition (2.6) (of course, there is no guarantee at this stage that there exists $\delta_\Lambda W$ satisfying eq. (2.6)). Since (2.5) should hold for arbitrary W, W can be regarded as arbitrariness in the solution of $Q_{\mathrm{B}} B = 0$ just like the collective coordinates of soliton solutions in conventional field theories.

Although in the complete treatment W is expected to represent the same number of degrees of freedom as Dirichlet open strings, we have not yet succeeded in developing a systematic method of treating within our framework all the degrees of freedom associated with D-brane. In this section, we shall consider the simplest case of taking as W only the *constant* field strength $F_{\mu\nu}$ of the massless gauge field on the D-p-brane. In Sec. 4 we shall add to W the degrees of freedom representing the tilt of the D-brane. Let $\mu = 0, 1, \cdots, p$ and $i = p+1, \cdots, d-1$ denote the space-time indices parallel and perpendicular to the p-brane, respectively. Then, the state $|B(F)\rangle$ satisfying the BRST-invariance condition (2.5) has been known as the boundary state [8, 9, 10, 11, 12]:

$$\left| B(F)(x^M, \overline{c}_0, \widetilde{\alpha}) \right\rangle = N(F) \left| B_{\mathrm{N}}(F) \right\rangle \otimes \left| B_{\mathrm{D}} \right\rangle \otimes \left| B_{\mathrm{gh}} \right\rangle, \tag{2.7}$$

with the factor states $|B_{\mathrm{N,D,gh}}\rangle$ given by[†]

$$|B_{\mathrm{N}}(F)\rangle = \exp \left\{ -\sum_{n \geq 1} \frac{1}{n} \alpha_{-n}^{(+)\mu} \mathcal{O}(F)_\mu{}^\nu \alpha_{-n\nu}^{(-)} \right\} |0\rangle_{p+1}, \tag{2.8}$$

$$|B_{\mathrm{D}}\rangle = \exp \left\{ \sum_{n \geq 1} \frac{1}{n} \alpha_{-n}^{(+)i} \alpha_{-ni}^{(-)} \right\} |0\rangle_{d-p-1} \, \delta^{d-p-1}(x^i), \tag{2.9}$$

$$|B_{\mathrm{gh}}\rangle = \exp \left\{ \sum_{n \geq 1} \left(c_{-n}^{(+)} \overline{c}_{-n}^{(-)} + c_{-n}^{(-)} \overline{c}_{-n}^{(+)} \right) \right\} |0\rangle_{\mathrm{gh}}. \tag{2.10}$$

In eq. (2.8), $\mathcal{O}$ is a $(p+1) \times (p+1)$ matrix satisfying the orthonormality condition

$$\mathcal{O}_\mu{}^\rho \eta_{\rho\lambda} \mathcal{O}_\nu{}^\lambda = \eta_{\mu\nu}, \tag{2.11}$$

[†] The Lorentz indices $M = (\mu, i)$ are raised/lowered by using the flat metric $\eta^{MN} = \eta_{MN} = (\eta_{\mu\nu}, \delta_{ij}) = \mathrm{diag}(-1, 1, \cdots, 1)$.

and is expressed in terms of an anti-symmetric constant matrix $F_{\mu\nu}$ as

$$\mathcal{O}(F)_\mu{}^\nu \equiv \left[(1+F)^{-1}(1-F) \right]_\mu^{\ \nu}. \tag{2.12}$$

The state $|B(F)\rangle$ is characterized by the following conditions for the string coordinates,

$$X^i(\sigma)\,|B(F)\rangle = 0, \tag{2.13}$$

$$\left(P_\mu(\sigma) - F_{\mu\nu}\frac{d}{d\sigma}X^\nu(\sigma) \right)|B(F)\rangle = 0, \tag{2.14}$$

$$\pi_c(\sigma)\,|B(F)\rangle = \pi_{\bar{c}}(\sigma)\,|B(F)\rangle = 0. \tag{2.15}$$

They are equivalently expressed in terms of the oscillation modes as

$$\left(\alpha_n^{(+)\,i} - \alpha_{-n}^{(-)\,i} \right)|B(F)\rangle = 0, \tag{2.16}$$

$$\left(\alpha_{n\,\mu}^{(+)} + \mathcal{O}_\mu{}^\nu \alpha_{-n\,\nu}^{(-)} \right)|B(F)\rangle = \left(\alpha_n^{(-)\,\mu} + \alpha_{-n}^{(+)\,\nu}\mathcal{O}_\nu{}^\mu \right)|B(F)\rangle = 0, \tag{2.17}$$

$$\left(c_n^{(+)} + c_{-n}^{(-)} \right)|B(F)\rangle = \left(\bar{c}_n^{(+)} - \bar{c}_{-n}^{(-)} \right)|B(F)\rangle = 0, \tag{2.18}$$

$$\left(x^i, \frac{\partial}{\partial x^\mu}, \frac{\partial}{\partial \bar{c}_0} \right)|B(F)\rangle = 0, \tag{2.19}$$

with $n = \pm 1, \pm 2, \cdots$. The BRST invariance of $|B(F)\rangle$,

$$Q_{\mathrm{B}}\,|B(F)\rangle = 0, \tag{2.20}$$

may be understood from the form of Q_{B} (see eq. (A.8) for the complete expression),

$$Q_{\mathrm{B}} = 2\sqrt{\pi}\int_0^{2\pi} d\sigma \left\{ i\pi_{\bar{c}}\left[\cdots\right] - c\left(P_M X'^M + c'\pi_c + \pi'_{\bar{c}}\bar{c} \right) \right\}, \tag{2.21}$$

and the properties (2.13), (2.14) and (2.15), and in particular, the anti-symmetric nature of $F_{\mu\nu}$.[‡]

The front factor $N(F)$ in eq. (2.7) cannot be determined at this stage from the requirement of the BRST-invariance (2.20) alone. In the next subsection we shall fix $N(F)$ using the gauge invariance requirement. The resultant $N(F)$ will agree with the one obtained previously from quite a different argument [11].

[‡] Although the normal ordering for Q_{B} is ignored in this argument based on eq. (2.21), the part of Q_{B} where the normal ordering is relevant is $L\left(\partial/\partial\bar{c}_0\right)$ (see (A.10)), which annihilates $|B(F)\rangle$ since it is independent of $\bar{c}_0$.

2.2 Determination of $N(F)$ and the transformation law of $F_{\mu\nu}$

In this subsection we shall examine eq. (2.6), the condition to determine $\delta_\Lambda W$. Since we have restricted W to a subspace of *constat* field strength F, we are not allowed to consider arbitrary gauge transformation functional Λ: $\delta_\Lambda W$ is in general not confined to our subspace. As one of the "allowed" Λ, let us consider the following Λ_-:

$$|\Lambda_-(x, \overline{c}_0, \widetilde{\alpha})\rangle = i\,\overline{c}_0 \left(\alpha_{-1}^{(+)\mu} \overline{c}_{-1}^{(-)} - \alpha_{-1}^{(-)\mu} \overline{c}_{-1}^{(+)} \right) |0\rangle\, \zeta_\mu(x, \widetilde{\alpha}), \qquad (2.22)$$

with ζ_μ linear in x^μ,

$$\zeta_\mu(x, \widetilde{\alpha}) = a_{\mu\nu} x^\nu. \qquad (2.23)$$

The gauge transformation (2.2) for this Λ_- induces the shift $\delta_{\Lambda_-} B_{\mu\nu} = \partial_\mu \zeta_\nu - \partial_\nu \zeta_\mu + \ldots$ on massless anti-symmetric tensor $B_{\mu\nu}$ contained in Φ (see Sec. 5).

Since Λ_- (2.22) does not depend on $\widetilde{\alpha}$, implying that it has vanishing string-length $\alpha = 0$, the star product $\Psi * \Lambda_-$ for any string functional Ψ is expressed as

$$|\Psi * \Lambda_-\rangle = \frac{1}{2} a_{\mu\nu} \mathcal{D}_-^{\mu\nu} |\Psi\rangle, \qquad (2.24)$$

in terms of the linear anti-hermitian operator $\mathcal{D}_-^{\mu\nu}$ given by

$$\begin{aligned}
\mathcal{D}_-^{\mu\nu} &= -i \int_0^{2\pi} d\sigma\, \frac{dX^\mu(\sigma)}{d\sigma} X^\nu(\sigma) - \eta^{\mu\nu} \int_0^{2\pi} d\sigma\, i\pi_c(\sigma)|_{\text{oscl}} \cdot i\pi_{\overline{c}}(\sigma)|_{\text{oscl}} \\
&= -\frac{1}{2} \sum_{n \geq 1} \frac{1}{n} \left(\alpha_{-n}^{(+)\mu} \alpha_n^{(+)\nu} - \alpha_{-n}^{(-)\mu} \alpha_n^{(-)\nu} + \alpha_n^{(+)\mu} \alpha_n^{(-)\nu} - \alpha_{-n}^{(+)\mu} \alpha_{-n}^{(-)\nu} - [\mu \leftrightarrow \nu] \right) \\
&\quad + (\text{ghost coordinates part}), \qquad\qquad\qquad\qquad (2.25)
\end{aligned}$$

where $i\pi_c(\sigma)|_{\text{oscl}}$ denotes the non-zero mode part of $i\pi_c(\sigma)$. Details of the derivation of (2.24) is presented in Appendix B. The first term of $\mathcal{D}_-^{\mu\nu}$, $-i \int_0^{2\pi} d\sigma\, (dX^\mu/d\sigma)\, \zeta_\mu(X)$, is a geometrically natural one. We do not know the intuitive interpretation of the ghost coordinate part of $\mathcal{D}_-^{\mu\nu}$.

Applying eq. (2.24) for $\Psi = B(F)$, we obtain

$$\begin{aligned}
|B(F) * \Lambda_-\rangle = \Bigg\{ &-\frac{1}{2} \zeta(0)\, \text{tr}\left[\left(a - a^T \right) \frac{1}{1 + F} \right] \\
&+ \sum_{n \geq 1} \frac{1}{n} \alpha_{-n}^{(+)\mu} \left[\frac{1}{1 + F} \left(a - a^T \right) \frac{1}{1 + F} \right]_{\mu\nu} \alpha_{-n}^{(-)\nu} \Bigg\} |B(F)\rangle, \quad (2.26)
\end{aligned}$$

where a^T is the transposition of matrix a, and $\zeta(0)$ is the value of the zeta-function at the origin; $\zeta(0) = \sum_{n=1}^\infty 1$. In deriving eq. (2.26) we have used the fact that $|B(F)\rangle$ is annihilated by the ghost coordinates part of $\mathcal{D}_-^{\mu\nu}$ due to (2.15).

We would like to determine the transformation law $\delta_{\Lambda_-} F$ of the (constant) field strength F under the present gauge transformation. $\delta_{\Lambda_-} F$ should satisfy (2.6), namely,

$$\delta_{\Lambda_-} |B(F)\rangle = 2 |B(F) * \Lambda_-\rangle. \tag{2.27}$$

It is easily seen that, if such a $\delta_{\Lambda_-} F$ exists, it should be given by

$$\delta_{\Lambda_-} F_{\mu\nu} = \partial_\nu \zeta_\mu - \partial_\mu \zeta_\nu = a_{\mu\nu} - a_{\nu\mu}, \tag{2.28}$$

since the $\alpha_{-n}^{(+)} \alpha_{-n}^{(-)}$ term on the RHS of (2.26) is nothing but the variation of the exponent of $|B_{\mathrm{N}}(F)\rangle$ under this δ_{Λ_-} (2.28).

In order for eq. (2.27) be satisfied completely, the first term on the RHS of eq. (2.26) must be equal to $\delta_{\Lambda_-} \ln N(F)$:

$$\delta_{\Lambda_-} \ln N(F) = -\zeta(0) \, \mathrm{tr} \left[\left(a - a^T \right) \frac{1}{1+F} \right], \tag{2.29}$$

This fixes $N(F)$ to be

$$N(F) = \frac{T_p}{4} \left[\det \left(1 + F \right) \right]^{-\zeta(0)}, \tag{2.30}$$

where T_p is a constant, and the factor $1/4$ is for the convenience of the comparison with the σ-model approach in Sec. 5. Thus we have determined $\delta_{\Lambda_-} F$ as well as the front factor $N(F)$ which satisfy eq. (2.27). The form of the front factor (2.30) agrees with the one determined by different arguments [11].

2.3 Born-Infeld action

One might think that the invariance of the system $S_0 + S_{\mathrm{source}}$ under the gauge transformation δ_{Λ_-} has been established since we have eqs. (2.20) and (2.27). However, this is not the case due to the fact that $Q_{\mathrm{B}} B(F) = 0$ does not ensure $B(F) \cdot \overrightarrow{Q_{\mathrm{B}}} \Lambda_- = 0$ for the present Λ_- (the right-arrow over Q_{B} indicates that it should operate on the right). In fact, we have

$$B(F) \cdot \overrightarrow{Q_{\mathrm{B}}} \Lambda_- = \int dz_0 \, \langle B(F)| \overrightarrow{Q_{\mathrm{B}}} |\Lambda_-\rangle = 2 V_{p+1} V_{\widetilde{\alpha}} \, \mathrm{tr} \left[\left(a - a^T \right) \frac{1}{1+F} \right] N(F) \neq 0, \tag{2.31}$$

where $V_{p+1} = \int d^{p+1} x$ is the space-time volume of the D-p-brane and $V_{\widetilde{\alpha}} = \int d\widetilde{\alpha}$ is the volume of the $\widetilde{\alpha}$-space. The impossibility of reversing the direction of the arrow over Q_{B} in (2.31) may be explained as follows. Note that the part of Q_{B} contributing to $Q_{\mathrm{B}} |\Lambda_-\rangle$ is

$i \sum_{\pm} \left(c^{(\pm)}_{-1} \alpha^{(\pm)\nu}_1 + c^{(\pm)}_1 \alpha^{(\pm)\nu}_{-1} \right) \partial/\partial x^\mu$. Since $\Lambda_- \propto x$, we have, extracting the x-part of the inner product,

$$B(F) \cdot \overrightarrow{Q_{\mathrm B}} \Lambda_- \sim \int dx1 \frac{\overrightarrow{\partial}}{\partial x} x \neq - \int dx1 \frac{\overleftarrow{\partial}}{\partial x} x \sim B(F) \cdot \overleftarrow{Q_{\mathrm B}} \Lambda_- = 0. \tag{2.32}$$

To achieve the full gauge invariance under δ_{Λ_-}, we have to add to $S_0 + S_{\mathrm{source}}$ a purely F term $I(F)$ whose δ_{Λ_-} transformation cancels (2.31):

$$\delta_{\Lambda_-} I(F) + B(F) \cdot \overrightarrow{Q_{\mathrm B}} \Lambda_- = 0. \tag{2.33}$$

In view of eq. (2.29), the desired $I(F)$ satisfying eq. (2.33) is seen to be given by

$$I(F) = \frac{2V_{p+1}V_{\widetilde\alpha}}{\zeta(0)} N(F) = \frac{2}{\zeta(0)} \int d^{p+1}x \int d\widetilde\alpha \, N(F). \tag{2.34}$$

This is nothing but the Born-Infeld action if we adopt the zeta-function regularization $\zeta(0) = -1/2$.

2.4 Total action

Summarizing the results of the previous subsections, the final expression of our closed SFT system coupled to a D-brane is described by the action $S_{\mathrm{tot}}[\Phi, F]$,

$$S_{\mathrm{tot}}[\Phi, F] = S_0[\Phi] + B(F) \cdot \Phi + I(F). \tag{2.35}$$

S_{tot} is invariant under the gauge transformation δ_{Λ_-} with Λ_- given by (2.22) and (2.23). The transformation rules of Φ and F are given respectively by eqs. (2.2) and (2.28).

3 Linear coordinate transformation

As another gauge transformation which is closed within the constant $F_{\mu\nu}$, let us consider the one corresponding to the linear coordinate transformation, $x^\mu \to x^\mu - \xi^\mu(x, \widetilde\alpha)$ with

$$\xi^\mu(x, \widetilde\alpha) = b^\mu{}_\nu x^\nu. \tag{3.1}$$

Such a coordinate transformation is generated by

$$|\Lambda_+(x, \overline c_0, \widetilde\alpha)\rangle = i\,\overline c_0 \left(\alpha^{(+)}_{-1\,\mu} \overline c^{(-)}_{-1} + \alpha^{(-)}_{-1\,\mu} \overline c^{(+)}_{-1} \right) |0\rangle \, \xi^\mu(x, \widetilde\alpha), \tag{3.2}$$

which is symmetric with respect to the left- and the right-moving oscillators. Similarly to (2.24), we have for any Ψ (see Appendix B),

$$|\Psi * \Lambda_+\rangle = \frac{1}{2} b_{\mu\nu} \mathcal{D}_+^{\mu\nu} |\Psi\rangle,$$ (3.3)

where $\mathcal{D}_+^{\mu\nu}$ is a linear operator given by

$$\mathcal{D}_+^{\mu\nu} = \frac{i}{2} \int_0^{2\pi} d\sigma \{X^\nu(\sigma), P^\mu(\sigma)\} - \eta^{\mu\nu} \left(\frac{1}{2} \left\{ \widetilde{\alpha}, \frac{\partial}{\partial\widetilde{\alpha}} \right\} + \mathcal{G} \right).$$ (3.4)

On the RHS of eq. (3.4), the oscillator expression of the first term is

$$\frac{i}{2} \int_0^{2\pi} d\sigma \{X^\nu(\sigma), P^\mu(\sigma)\} = \frac{1}{2} \left\{ x^\nu, \frac{\partial}{\partial x_\mu} \right\}$$
$$- \frac{1}{2} \sum_\pm \sum_{n \geq 1} \frac{1}{n} \left(\alpha_{-n}^{(\pm)\mu} \alpha_n^{(\pm)\nu} - \alpha_{-n}^{(\pm)\nu} \alpha_n^{(\pm)\mu} + \alpha_n^{(\pm)\mu} \alpha_n^{(\mp)\nu} - \alpha_{-n}^{(\pm)\mu} \alpha_{-n}^{(\mp)\nu} \right),$$ (3.5)

while the second term $\mathcal{G}$ consists solely of the ghost coordinates:

$$\mathcal{G} = \frac{1}{2} \int_0^{2\pi} d\sigma \left[\overline{c}|_{\text{oscl}}(\sigma), i\pi_{\overline{c}}|_{\text{oscl}}(\sigma) \right] + \widetilde{N}_{\text{gh}}$$
$$= \frac{1}{2} \sum_\pm \sum_{n \geq 1} \left(\overline{c}_n^{(\pm)} c_n^{(\mp)} + \overline{c}_{-n}^{(\pm)} c_{-n}^{(\mp)} \right) + \frac{1}{2} \widetilde{N}_{\text{gh}},$$ (3.6)

with $\widetilde{N}_{\text{gh}}$ being the oscillator part of the ghost number operator (A.15). Note that the first term of $\mathcal{D}_+^{\mu\nu}$, $(1/2) \int_0^{2\pi} d\sigma \{X^\nu, \delta/\delta X_\mu\}$, is in fact the operator of linear coordinate transformation. For $b^\mu{}_\nu$ with non-vanishing trace, $b_{\mu\nu} \mathcal{D}_+^{\mu\nu}$ also contains the $\widetilde{\alpha}$ and the ghost coordinate parts, whose geometrical interpretation is not clear to us.

Applying eq. (3.3) to $\Psi = B(F)$, we have

$$2 |B(F) * \Lambda_+\rangle = \left\{ \frac{1}{2} \zeta(0) \, \text{tr} \left[\left(b + b^T \right) \left(\frac{1 - F}{1 + F} + 1 \right) \right] \right.$$
$$\left. + 2 \sum_{n \geq 1} \frac{1}{n} \alpha_{-n}^{(+)\mu} \left[\frac{1}{1 + F} \left(b^T F + Fb \right) \frac{1}{1 + F} \right]_{\mu\nu} \alpha_{-n}^{(-)\nu} \right\} |B(F)\rangle.$$ (3.7)

In deriving eq. (3.7), we have used in particular that

$$\mathcal{G} |B(F)\rangle = -\zeta(0) |B(F)\rangle,$$ (3.8)

and that the zero-mode part of $\mathcal{D}_+^{\mu\nu}$,

$$\mathcal{D}_+^{\mu\nu}\big|_{0-\text{modes}} = \frac{1}{2} \left\{ x^\nu, \frac{\partial}{\partial x_\mu} \right\} - \frac{1}{2} \eta^{\mu\nu} \left\{ \widetilde{\alpha}, \frac{\partial}{\partial\widetilde{\alpha}} \right\} = x^\nu \frac{\partial}{\partial x_\mu} - \eta^{\mu\nu} \widetilde{\alpha} \frac{\partial}{\partial\widetilde{\alpha}},$$ (3.9)

9

annihilates $|B(F)\rangle$ since it depends on neither x^μ nor $\widetilde\alpha$ for a constant F.

Our next task is to identify the transformation rule of F under the present gauge transformation δ_{Λ_+}. The equation for determining $\delta_{\Lambda_+} F$ is

$$\delta_{\Lambda_+} |B(F)\rangle = 2\,|B(F) * \Lambda_+\rangle. \tag{3.10}$$

Since δ_{Λ_+} corresponds to the linear coordinate transformation, it is natural to take

$$\delta_{\Lambda_+} F_{\mu\nu} = \partial_\mu \xi^\lambda F_{\lambda\nu} + \partial_\nu \xi^\lambda F_{\mu\lambda} + \xi^\lambda \partial_\lambda F_{\mu\nu} = b^\lambda{}_\mu F_{\lambda\nu} + b^\lambda{}_\nu F_{\mu\lambda}, \tag{3.11}$$

or equivalently $\delta_{\Lambda_+} F = b^T F + F b$ in matrix notation. However, since under (3.11) we have

$$
\delta_{\Lambda_+} |B(F)\rangle = \left\{ \frac{1}{2}\,\zeta(0)\,\mathrm{tr}\left[\left(b + b^T\right)\left(\frac{1-F}{1+F} - 1\right)\right] \right.
$$
$$
\left. + 2\sum_{n\geq 1}\frac{1}{n}\alpha^{(+)\mu}_{-n}\left[\frac{1}{1+F}\left(b^T F + F b\right)\frac{1}{1+F}\right]_{\mu\nu}\alpha^{(-)\nu}_{-n} \right\}|B(F)\rangle, \tag{3.12}
$$

eq. (3.10) holds only for a traceless b satisfying $b^\mu{}_\mu = 0$ due to disagreement between the oscillator independent terms of eqs. (3.7) and (3.12).[§] Note that the first term on the RHS of (3.12) is the contribution of $\delta_{\Lambda_+} \ln N(F)$:

$$\delta_{\Lambda_+} \ln N(F) = -\zeta(0)\,\mathrm{tr}\left(\frac{1}{1+F}\,\delta_{\Lambda_+} F\right) = \frac{1}{2}\,\zeta(0)\,\mathrm{tr}\left[\left(b + b^T\right)\left(\frac{1-F}{1+F} - 1\right)\right]. \tag{3.13}$$

To establish the invariance under δ_{Λ_+}, we have to confirm also

$$\delta_{\Lambda_+} I(F) + B(F) \cdot \overrightarrow{Q_{\mathrm{B}}}\Lambda_+ = 0, \tag{3.14}$$

for $I(F)$ of eq. (2.34). (Note that, since Λ_+ is proportional to x, reversing the direction of the operation of Q_{B} in the inner product $B(F) \cdot \overrightarrow{Q_{\mathrm{B}}}\Lambda_+$ is not allowed as in the case of Λ_-.) Using eq. (3.13) and

$$B(F) \cdot \overrightarrow{Q_{\mathrm{B}}}\Lambda_+ = \int dz_0\,\langle B(F)|\,\overrightarrow{Q_{\mathrm{B}}}\,|\Lambda_+\rangle = -V_{p+1}\,V_{\widetilde\alpha}\,\mathrm{tr}\left[\left(b + b^T\right)\left(\frac{1-F}{1+F} + 1\right)\right]N(F), \tag{3.15}$$

we find that eq. (3.14) holds for traceless b.

[§] The tracelessness restriction $b^\mu{}_\mu = 0$ persists even if we take into account the change of the measure dz_0 since the variations of $d^{p+1}x$ and $d\widetilde\alpha$ cancel each other as seen from eq. (3.9).

4 Tilting the D-brane

So far we have considered a D-p-brane fixed at $x^i = 0$ ($i = p+1, \cdots, d-1$). In this section we shall allow the D-brane to "tilt", namely, consider a D-brane

$$x^i = \theta^i_\mu x^\mu, \tag{4.1}$$

specified by θ^i_μ (θ^i_μ with $\mu \neq 0$ is the tilt angle of the D-brane and θ^i_0 is its velocity). The boundary state $|B(F, \theta)\rangle$ corresponding to such a tilted D-brane is obtained from $|B(F)\rangle$ of the previous sections as[¶]

$$|B(F, \theta)\rangle = U(\theta) |B(F)\rangle, \tag{4.2}$$

where $U(\theta)$ is a unitary operator,

$$U(\theta) = \exp\left(-\theta^i_\mu \mathcal{D}_{+i}{}^\mu\right), \tag{4.3}$$

with $\mathcal{D}_{+i}{}^\mu$ given by

$$\mathcal{D}_{+i}{}^\mu = i \int_0^{2\pi} d\sigma X^\mu(\sigma) P_i(\sigma). \tag{4.4}$$

In fact, $U(\theta)$ effects the following transformation on the string coordinates and their conjugates,

$$U(\theta) \begin{pmatrix} X^\mu \\ P_\mu \\ X^i \\ P_i \end{pmatrix} U(\theta)^{-1} = \begin{pmatrix} X^\mu \\ P_\mu + \theta^j_\mu P_j \\ X^i - \theta^i_\nu X^\nu \\ P_i \end{pmatrix}, \tag{4.5}$$

and hence $|B(F, \theta)\rangle$ (4.2) satisfies, instead of eqs. (2.13) and (2.14), the following two:

$$\left(X^i(\sigma) - \theta^i_\nu X^\nu(\sigma)\right) |B(F, \theta)\rangle = 0, \tag{4.6}$$

$$\left(P_\mu(\sigma) + \theta^j_\mu P_j(\sigma) - F_{\mu\nu}(dX^\nu(\sigma)/d\sigma)\right) |B(F, \theta)\rangle = 0. \tag{4.7}$$

Our new boundary state $|B(F, \theta)\rangle$ is also BRST invariant:

$$Q_{\mathrm{B}} |B(F, \theta)\rangle = 0. \tag{4.8}$$

This is easily seen by noting that $P_M(\sigma) X'^M(\sigma)$ contained in Q_{B} is invariant under the transformation of $U(\theta)$ (recall eq. (2.21)):

$$U(\theta) P_M X'^M U(\theta)^{-1} = P_M X'^M. \tag{4.9}$$

[¶] Note that $|B(F)\rangle$ itself can be expressed as $|B(F)\rangle = \exp\left(\frac{1}{2} F_{\mu\nu} \mathcal{D}_-^{\mu\nu}\right) |B(F=0)\rangle$.

We would like to repeat the construction of "closed SFT + D-brane" system of Sec. 2 by taking $F_{\mu\nu}$ and θ^i_μ as dynamical variables associated with the D-brane. The total action of this should be given by

$$S_{\text{tot}}[\Phi, F, \theta] = S_0[\Phi] + B(F, \theta) \cdot \Phi + I(F, \theta), \tag{4.10}$$

and we shall determine $I(F, \theta)$ and a possible (F, θ)-dependent factor multiplied on $|B(F, \theta)\rangle$ so that the system has gauge invariance as before.

Let us consider the gauge transformation δ_{Λ_-}. $B(F, \theta)$ and $I(F, \theta)$ have to satisfy the following two conditions:

$$\delta_{\Lambda_-}|B(F, \theta)\rangle = 2|B(F, \theta) * \Lambda_-\rangle = a_{\mu\nu}\mathcal{D}^{\mu\nu}_-|B(F, \theta)\rangle, \tag{4.11}$$

$$\delta_{\Lambda_-}I(F, \theta) + B(F, \theta) \cdot \overrightarrow{Q_{\text{B}}}\Lambda_- = 0. \tag{4.12}$$

From (4.11) we find that the transformation rule of F is as before and θ is inert under δ_{Λ_-},

$$\delta_{\Lambda_-}F_{\mu\nu} = a_{\mu\nu} - a_{\nu\mu}, \quad \delta_{\Lambda_-}\theta^i_\mu = 0, \tag{4.13}$$

since $\mathcal{D}^{\mu\nu}_-$ (2.25) commutes with $U(\theta)$, $[U(\theta), \mathcal{D}^{\mu\nu}_-] = 0$. We need no extra (F, θ)-dependent factor multiplying $|B(F, \theta)\rangle$ of eq. (4.2).

Our next task is the determination of $I(F, \theta)$ satisfying (4.12). We shall do this in a manner different from Sec. 2. For this purpose, observe that

$$\overline{c}_0 \langle 0|\, a_{\mu\nu}\mathcal{D}^{\mu\nu}_-|B(F, \theta)\rangle = -\frac{1}{2}\zeta(0)\langle\Lambda_-|\overleftarrow{Q_{\text{B}}}|B(F, \theta)\rangle. \tag{4.14}$$

This is easily understood by noticing that $\mathcal{D}^{\mu\nu}_-$ and the exponent of $|B(F, \theta)\rangle$ are given as sums over the oscillator level number n, and that each term in the n-summation in $\mathcal{D}^{\mu\nu}_-$ (2.25) gives equal contribution to the LHS of (4.14). Then, comparing $\langle 0|$ eq. (4.11)$\rangle$ and (4.12), we find that the desired $I(F, \theta)$ is given by

$$I(F, \theta) = \frac{2}{\zeta(0)}\int d^{d+1}x \int d\widetilde{\alpha}\,\langle 0|B(F, \theta)\rangle. \tag{4.15}$$

To calculate the inner product $\langle 0|B(F, \theta)\rangle = \langle 0|\,U(\theta)\,|B(F)\rangle$, let us express $U(\theta)$ (4.3) as

$$U(\theta) = e^{-i\theta^i_\mu x^i p_\mu}e^{-u^\dagger(\theta)}e^{u(\theta)}, \tag{4.16}$$

with $u(\theta)$ given by (note that $\left[u(\theta), u^\dagger(\theta)\right] = 0$)

$$u(\theta) = \frac{1}{2}\theta^i_\mu\sum_{n\geq 1}\frac{1}{n}\left(\alpha^{(+)}_{-n} + \alpha^{(-)}_n\right)_i\left(\alpha^{(+)}_n - \alpha^{(-)}_{-n}\right)^\mu. \tag{4.17}$$

Then, using eqs. (2.16) and (2.17) to express the annihilation operators in $u(\theta)$ in terms of the creation ones and making use of the formula,

$$\langle 0| \exp\left(\frac{1}{2}a_a M_{ab} a_b\right) \exp\left(\frac{1}{2}a_a^\dagger N_{ab} a_b^\dagger\right)|0\rangle = \left[\det\left(1 - NM\right)\right]^{-1/2}, \tag{4.18}$$

valid for creation/annihilation operators $(a_a^\dagger, a_a)$ with $[a_a, a_b^\dagger] = \delta_{a,b}$, we obtain

$$\langle 0|B(F,\theta)\rangle = N(F) \left[\det\left(\delta_\nu^\mu + \frac{1}{2}\left(\eta^{\mu\lambda} + \mathcal{O}^{\mu\lambda}\right)\theta_\lambda^i \theta_\nu^i\right)\right]^{-\zeta(0)} \delta^{d-p-1}\left(x^i - \theta_\mu^i x^\mu\right). \tag{4.19}$$

Therefore, $I(F,\theta)$ is given by

$$I(F,\theta) = \frac{T_p}{2\zeta(0)} \int d^{p+1}x \int d\tilde{\alpha}\left[-\det\left(\eta_{\mu\nu} + \theta_\mu^i \theta_\nu^i + F_{\mu\nu}\right)\right]^{-\zeta(0)}. \tag{4.20}$$

For the use in the next section, we also present a more explicit expression of $|B(F,\theta)\rangle$:

$$|B(F,\theta)\rangle = N\left(F + \theta\theta\right)\exp\left\{\sum_{n\geq 1}\frac{1}{n}\alpha_{-n}^{(+)M} A_{MN}\alpha_{-n}^{(-)N}\right\}|0\rangle_d \otimes |B_{\mathrm{gh}}\rangle \,\delta^{d-p-1}\left(x^i - \theta_\mu^i x^\mu\right), \tag{4.21}$$

where $F + \theta\theta$ is short for $F_{\mu\nu} + \theta_\mu^i \theta_\nu^i$, and the $d \times d$ matrix A_{MN} is given by

$$A_{\mu\nu} = \eta_{\mu\nu} - \left(2/\left(\tilde{\eta} + F\right)\right)_{\mu\nu}, \quad A_{\mu i} = -\left(2/\left(\tilde{\eta} + F\right)\right)_\mu{}^\lambda \theta_\lambda^i,$$
$$A_{i\mu} = -\theta_\lambda^i \left(2/\left(\tilde{\eta} + F\right)\right)^\lambda{}_\mu, \quad A_{ij} = \delta_{ij} - \theta_\mu^i \left(2/\left(\tilde{\eta} + F\right)\right)^{\mu\nu}\theta_\nu^j, \tag{4.22}$$

with

$$\tilde{\eta}(\theta)_{\mu\nu} = \eta_{\mu\nu} + \theta_\mu^i \theta_\nu^i. \tag{4.23}$$

$|B(F,\theta)\rangle$ of (4.21) has correct normalization and satisfies the conditions (4.6) and (4.7).

Here, we have considered only the gauge transformation δ_{Λ_-}. It is straightforward to confirm the invariances under other gauge transformations closed within constant $F_{\mu\nu}$ and θ_μ^i. For example, under the transformation δ_{Λ_+} of Sec. 3, $F_{\mu\nu}$ and θ_μ^i should transform as $\delta_{\Lambda_+}F_{\mu\nu} = b^\lambda{}_\mu F_{\lambda\nu} + b^\lambda{}_\nu F_{\mu\lambda}$ and $\delta_{\Lambda_+}\theta_\mu^i = b^\nu{}_\mu \theta_\nu^i$, respectively. Furthermore, the introduction of θ_μ^i allows the gauge transformation δ_{Λ_t} generated by

$$|\Lambda_t\rangle = i\,\bar{c}_0\left(\bar{c}_{-1}^{(+)}\alpha_{-1\,i}^{(-)} + \bar{c}_{-1}^{(-)}\alpha_{-1\,i}^{(+)}\right)|0\rangle\, b_\mu^i x^\mu. \tag{4.24}$$

For this Λ_t we have $|\Psi * \Lambda_t\rangle = \frac{1}{2}\theta_\mu^i \mathcal{D}_{+i}{}^\mu |\Psi\rangle$, and the action $S_{\mathrm{tot}}[\Phi, F, \theta]$ (4.10) is invariant under δ_{Λ_t} if the transformation law of $(F_{\mu\nu}, \theta_\mu^i)$ is defined by $\delta_{\Lambda_t}F_{\mu\nu} = 0$ and $\delta_{\Lambda_t}\theta_\mu^i = -b_\mu^i$.

5 Comparison with the σ-model approach

In this section we shall examine the correspondence between our "SFT + D-p-brane" system (4.10) and the σ-model approach [9, 10, 11]. We show that the actions in both the approaches coincide with each other to the first non-trivial orders in the expansion in powers of massless fields of closed string.

First, let us consider the σ-model approach. It has been known that the low energy dynamics of the system of closed string coupled to a D-p-brane is described by the following effective action [3],

$$S_{\text{eff}} = S_{\text{bulk}} + S_{\text{D}}, \tag{5.1}$$

with the bulk part S_{bulk} and the D-brane action S_{D} given respectively by

$$S_{\text{bulk}} = \frac{1}{g^2} \int d^d x \sqrt{-G} e^{-2D} \left\{ R + 4 \left(\nabla D \right)^2 - \frac{1}{12} H^2 \right\}, \tag{5.2}$$

$$S_{\text{D}} = -T_p \int d^{p+1}\sigma \, e^{-D} \sqrt{-\det \left(\widetilde{G}_{\mu\nu} + \widetilde{B}_{\mu\nu} + F_{\mu\nu} \right)}, \tag{5.3}$$

where G_{MN} and D are the metric and the dilaton fields, respectively, and H_{MNP} is the field strength of the anti-symmetric tensor B_{MN}, $H = dB$. In eq. (5.3), $\widetilde{G}_{\mu\nu}$ and $\widetilde{B}_{\mu\nu}$ are the induced fields on the D-p-brane parameterized by the coordinate σ^μ ($\mu = 0, \cdots, p$). Namely, letting $Y^M(\sigma)$ denote the D-brane space-time coordinate, the induced metric $\widetilde{G}_{\mu\nu}(\sigma)$ is

$$\widetilde{G}_{\mu\nu}(\sigma) = \partial_\mu Y^M(\sigma) \, \partial_\nu Y^N(\sigma) \, G_{MN} \left(Y(\sigma) \right). \tag{5.4}$$

The expression of $\widetilde{B}_{\mu\nu}$ is quite similar.

For comparing (5.1) with our SFT approach, let us make the Weyl rescaling,

$$G_{MN} \to e^{4D/(d-2)} G_{MN}, \tag{5.5}$$

under which the bulk action (5.2) is reduced to

$$S_{\text{bulk}} = \frac{1}{g^2} \int d^d x \sqrt{-G} \left\{ R - \frac{4}{d-2} \left(\nabla D \right)^2 - \frac{1}{12} e^{-8D/(d-2)} H^2 \right\}. \tag{5.6}$$

As for the D-brane action (5.3), we expanded it in powers of the massless fields associated with closed string; D, B_{MN} and the metric fluctuation h_{MN} defined for the Weyl rescaled G_{MN} by

$$G_{MN} = \eta_{MN} + h_{MN}. \tag{5.7}$$

14

Adopting the static gauge with $\sigma^\mu = x^\mu$, $Y^M(x)$ for the tilted D-brane (4.1) is

$$Y^\mu(x) = x^\mu, \quad Y^i(x) = \theta^i_\mu x^\mu, \tag{5.8}$$

and hence the induced metric $\widetilde{G}_{\mu\nu}$ is given by

$$\widetilde{G}_{\mu\nu} = \widetilde{\eta}_{\mu\nu} + \widetilde{h}_{\mu\nu}, \tag{5.9}$$

in terms of $\widetilde{\eta}_{\mu\nu}$ of (4.23) and $\widetilde{h}_{\mu\nu}$ defined by

$$\widetilde{h}_{\mu\nu} = h_{\mu\nu} + \theta^i_\mu h_{i\nu} + \theta^j_\nu h_{\mu j} + \theta^i_\mu \theta^j_\nu h_{ij}. \tag{5.10}$$

The induced $\widetilde{B}_{\mu\nu}$ is also given by (5.10) with h replaced by B. Then, keeping only the terms independent of and linear in (h_{MN}, D, B_{MN}), we have

$$S_D|_{\text{linear}} = -T_p \int d^{p+1}x \sqrt{-\det\left(\widetilde{\eta} + F\right)} \left\{ 1 - D + \frac{2D}{d-2} \,\mathrm{tr}\left(\frac{\widetilde{\eta}}{\widetilde{\eta}+F}\right) + \frac{1}{2}\,\mathrm{tr}\left(\frac{\widetilde{h}+\widetilde{B}}{\widetilde{\eta}+F}\right) \right\}. \tag{5.11}$$

Next, let us consider our SFT approach. The string field Φ consists of two parts, ϕ and ψ:

$$|\Phi\rangle = -\overline{c}_0 |\phi\rangle + |\psi\rangle. \tag{5.12}$$

They are expanded in terms of the component fields as follows (we keep only the massless fields):

$$|\phi(x)\rangle = \left\{ -\frac{1}{2}\hat{h}_{MN}(x)\left(\alpha^M_{-1}\alpha^N_{-1}\right)^{(+-)} + \frac{1}{2}B_{MN}(x)\left(\alpha^M_{-1}\alpha^N_{-1}\right)^{[+-]}, \right.$$
$$\left. -\hat{D}(x)\,(c_{-1}\overline{c}_{-1})^{(+-)} + f(x)\,(c_{-1}\overline{c}_{-1})^{[+-]} + \ldots \right\} |0\rangle \tag{5.13}$$

$$|\psi(x)\rangle = \frac{i}{2}\left\{ b_M(x)\left(\alpha^M_{-1}\overline{c}_{-1}\right)^{(+-)} + e_M(x)\left(\alpha^M_{-1}\overline{c}_{-1}\right)^{[+-]} + \ldots \right\}|0\rangle, \tag{5.14}$$

where we have used the (anti-)symmetrization symbol,

$$(ab)^{(+-)} \equiv a^{(+)}b^{(-)} + a^{(-)}b^{(+)}, \quad (ab)^{[+-]} \equiv a^{(+)}b^{(-)} - a^{(-)}b^{(+)}. \tag{5.15}$$

In this section we omit the $\widetilde{\alpha}$-dependence of the fields. Similarly, the component expansion of the gauge transformation functional Λ in (2.2) is given as

$$|\Lambda\rangle = i\overline{c}_0 \left\{ \xi_M(x)\left(\alpha^M_{-1}\overline{c}_{-1}\right)^{(+-)} + \zeta_M(x)\left(\alpha^M_{-1}\overline{c}_{-1}\right)^{[+-]} + \ldots \right\}|0\rangle$$
$$+ \eta(x)\,\overline{c}^{(+)}_{-1}\overline{c}^{(-)}_{-1}|0\rangle + \ldots. \tag{5.16}$$

15

In the following we shall consider only the lowest non-trivial parts of S_{tot} (2.35) in the power series expansion in the closed string massless component fields. Therefore, in S_0 (2.1) we keep only the kinetic term $(1/2g^2)\Phi \cdot Q_B\Phi$. Then, after integrating out the auxiliary fields b_M and e_M and gauging f away by using the gauge freedom of η in (5.16), we obtain

$$\frac{1}{2g^2}\,\Phi \cdot Q_B\Phi|_{\text{massless}} = \frac{1}{g^2}\int d^d x \left\{\left(\sqrt{-G}R\right)_{\text{quadratic}} - \frac{4}{d-2}(\partial D)^2 - \frac{1}{12}H_{MNP}H^{MNP}\right\},$$
(5.17)

where we have reexpressed $\hat{h}_{MN}$ and $\hat{D}$ in (5.13) in terms of new h_{MN} and D as

$$\hat{h}_{MN} = h_{MN} + \frac{4}{d-2}D\,\eta_{MN}, \quad \hat{D} = \frac{4}{d-2}D + \frac{1}{2}h_M{}^M.$$
(5.18)

For the first term on the RHS of (5.17), we have used the formula

$$\left(\sqrt{-G}R\right)_{\text{quadratic}} = \frac{1}{4}h^{MN}\left(\Box h_{MN} - 2\partial_N\partial^P h_{MP} + 2\partial_M\partial_N h_P^P - \eta_{MN}\Box h_P^P\right).$$
(5.19)

We see that eq. (5.17) coincides with the part of S_{bulk} (5.6) quadratic in the fluctuations (h_{MN}, D, B_{MN}).

Then, let us consider the D-p-brane parts of S_{tot} (4.10). Using eq. (4.21) and keeping only the massless component fields in Φ, we obtain

$$B(F,\theta) \cdot \Phi|_{\text{massless}} = 2\int d^{p+1}x\, N(F+\theta\theta)\left\{\frac{1}{2}\hat{h}_M{}^M - \text{tr}\left(\frac{\widetilde{\hat{h}} + \widetilde{B}}{\widetilde{\eta} + F}\right) - \hat{D}\right\}$$

$$= -4\int d^{p+1}x\, N(F+\theta\theta)\left\{-D + \frac{2D}{d-2}\text{tr}\left(\frac{\widetilde{\eta}}{\widetilde{\eta} + F}\right) + \frac{1}{2}\text{tr}\left(\frac{\widetilde{h} + \widetilde{B}}{\widetilde{\eta} + F}\right)\right\},$$
(5.20)

where $\widetilde{\hat{h}}_{\mu\nu}$ is defined by (5.10) with h replaced by $\hat{h}$. Adopting the zeta function regularization $\zeta(0) = -1/2$, we find that $B(F,\theta) \cdot \Phi|_{\text{massless}} + I(F,\theta)$ in SFT approach indeed coincides with $S_D|_{\text{linear}}$ (5.11) for a common T_p.

Finally, we shall mention the gauge transformation properties of the component fields. The massless component fields appearing in (5.17) transform under $\delta_\Lambda|_{\text{free}}\Phi \equiv Q_B\Lambda$ as

$$\delta_\Lambda|_{\text{free}}h_{MN} = \partial_M\xi_N + \partial_N\xi_M,$$
$$\delta_\Lambda|_{\text{free}}B_{MN} = \partial_M\zeta_N - \partial_N\zeta_M,$$
$$\delta_\Lambda|_{\text{free}}D = 0,$$
(5.21)

and the free action (5.17) is in fact invariant under (5.21). The transformation rule of the induced field $\widetilde{B}_{\mu\nu} = B_{\mu\nu} + \theta_\mu^i B_{i\nu} + \theta_\nu^j B_{\mu j} + \theta_\mu^i\theta_\nu^j B_{ij}$ under ζ_μ of (2.23) and $\zeta_i = 0$ is $\delta_{\Lambda_-}\widetilde{B}_{\mu\nu} =$

$\partial_\mu \zeta_\nu - \partial_\nu \zeta_\mu = a_{\nu\mu} - a_{\mu\nu}$ as is expected from the fact that $\widetilde{B}$ and $F_{\mu\nu}$ appear in S_D (5.3) in the combination $\widetilde{B}_{\mu\nu} + F_{\mu\nu}$.

We have seen the equivalence between our SFT approach and the σ-model approach to the first non-trivial orders in the expansion in powers of the massless closed string fields. To discuss the equivalence to higher orders, we have to carry out the integrations over the massive fields in our SFT approach.

6   Summary and discussions

We have constructed a system of closed SFT coupled to a D-brane on the basis of gauge invariance principle. Invariance under stringy local gauge transformation requires that the state B coupled to closed string field be annihilated by the BRST charge Q_B. The gauge invariance requirement also gives the equation which determines the transformation law of the dynamical variable W associated with the D-brane. Adopting as B the boundary state $B(F, \theta)$ which is a BRST invariant one, invariance requirement under a special gauge transformation which shifts the anti-symmetric tensor by a constant fixes the (F, θ)-dependence of the front factor of B as well as the gauge transformation law of the field strength $F_{\mu\nu}$ and the tilt angle θ^i_μ of the D-brane. Furthermore, due to the unboundedness of this gauge transformation at infinity, we need to introduce the Born-Infeld action to realize the gauge invariance. The invariance under linear coordinate transformation has also been studied. We have checked the correspondence between the action of our "closed SFT + D-brane" system and the effective action in the σ-model approach.

Our construction here is still incomplete and there are many subjects to be studied. One of the most important among them is to extend the dynamical variables associate with D-brane. In this paper, we considered only *constant* $F_{\mu\nu}$ and θ^i_μ. Since we know that D-branes have the same number of degrees of freedom as Dirichlet open string, we should be able to incorporate all of them in the present formalism. This extension includes generalizing constant $(F_{\mu\nu}, \theta^i_\mu)$ to x^μ-dependent ones $(A_\mu(x), Y^i(x))$, as well as introducing massive degrees of freedom on D-branes. One way of introducing non-constant $F_{\mu\nu}$ is to make use of gauge transformation. Assuming that the gauge transformation (2.22) with a general $\zeta_\mu(x)$ generates $\delta_{\Lambda_-} F_{\mu\nu}(x) = \partial_\mu \zeta_\nu(x) - \partial_\nu \zeta_\mu(x)$, we can determine the boundary state for an (infinitesimally) non-constant field strength $F_{\text{new}}(x) = F + d\zeta(x)$ as $B(F_{\text{new}}) = B(F) + 2B(F) * \Lambda_-$. Details of this extension will be given in a separate paper [25].

As another problem left in our formalism, we have to determine the D-brane tension T_p. In the world sheet approach, the D-brane tension has been determined by either using Lovelace-Fischler-Susskind mechanism [23, 24] or by comparing the one-loop vacuum energy of Dirichlet open string with the amplitude of massless field exchange in the low energy effective action S_{eff} (5.1) [3]. In our SFT approach, the boundary state B has been introduced as a state satisfying the BRST invariance condition (2.5), which is a linear equation and does not fix the absolute magnitude of B. It would be most interesting if we could "improve" our formalism in such a way that the boundary state is determined by a non-linear equation which allows the interpretation of the D-brane as a soliton in closed SFT.

Although we do not know how to determine the absolute value of T_p within our formalism, its dependence of the string coupling constant g can be deduced from the relation between the string coupling constant and the dilaton expectation value. This well-known relation is expressed in closed SFT as the property that S_0 (2.1) is invariant under following transformation of the string field Φ and the coupling constant g [26, 27, 22, 28, 29]:

$$\delta_{\text{D}} |\Phi\rangle = \left(\mathcal{D} + \frac{d-2}{2}\right) |\Phi\rangle + 2\sqrt{d-2}\, |\text{Dilaton}\rangle , \tag{6.1}$$

$$\delta_{\text{D}}g = \frac{d-2}{2}\, g, \tag{6.2}$$

where $\mathcal{D}$ is the dilatation operator defined by eq. (6) of ref. [29], and $|\text{Dilaton}\rangle$ is the zero momentum dilaton state. In our "closed SFT + D-brane" system, we can show that the total action S_{tot} of eqs. (2.35) and (4.10) is invariant under δ_{D} of eqs. (6.1) and (6.2) and suitably defined $\delta_{\text{D}}\left(F_{\mu\nu}, \theta_\mu^i\right)$, if $\delta_{\text{D}}T_p$ is given by

$$\delta_{\text{D}}T_p = -\frac{d-2}{2}T_p. \tag{6.3}$$

Eqs. (6.2) and (6.3) implies that $T_p \propto 1/g$.

Acknowledgments

18

Appendix

A Summary of the formulas in closed SFT

In this appendix we summarize various quantities in closed SFT used in the text.

String coordinates

String coordinates $\left(X^M(\sigma), c(\sigma), \overline{c}(\sigma)\right)$ and their conjugates $(P_M(\sigma), \pi_c(\sigma), \pi_{\overline{c}}(\sigma))$ are expanded in terms of the creation/annihilation operators as follows. For the space-time coordinate, we have

$$X^M(\sigma) = \frac{1}{\sqrt{\pi}}\left\{x^M + \frac{i}{2}\sum_{n\neq0}\frac{1}{n}\left(\alpha_n^{(+)M} - \alpha_{-n}^{(-)M}\right)e^{in\sigma}\right\}, \tag{A.1}$$

$$P_M(\sigma) = -i\frac{\delta}{\delta X^\mu(\sigma)} = \frac{1}{2\sqrt{\pi}}\left\{p_M + \sum_{n\neq0}\left(\alpha_n^{(+)M} + \alpha_{-n}^{(-)M}\right)e^{in\sigma}\right\}, \tag{A.2}$$

with $p_M = -i\partial/\partial x^M$. For the ghost coordinates,

$$\overline{c}(\sigma) = \frac{1}{2\sqrt{\pi}}\left\{\overline{c}_0 + \sum_{n\neq0}\left(\overline{c}_n^{(+)} + \overline{c}_{-n}^{(-)}\right)e^{in\sigma}\right\}, \tag{A.3}$$

$$i\pi_{\overline{c}}(\sigma) = \frac{\delta}{\delta\overline{c}(\sigma)} = \frac{1}{2\sqrt{\pi}}\left\{2\frac{\partial}{\partial\overline{c}_0} + \sum_{n\neq0}\left(c_n^{(+)} + c_{-n}^{(-)}\right)e^{in\sigma}\right\}, \tag{A.4}$$

$$c(\sigma) = -\frac{1}{2\sqrt{\pi}}\left\{i\frac{\partial}{\partial\pi_c^0} + \sum_{n\neq0}\left(c_n^{(+)} - c_{-n}^{(-)}\right)e^{in\sigma}\right\}, \tag{A.5}$$

$$i\pi_c(\sigma) = \frac{\delta}{\delta c(\sigma)} = -\frac{1}{2\sqrt{\pi}}\left\{-2i\pi_c^0 + \sum_{n\neq0}\left(\overline{c}_n^{(+)} - \overline{c}_{-n}^{(-)}\right)e^{in\sigma}\right\}. \tag{A.6}$$

The (anti-)commutation relations among $\left(\alpha_n^{(\pm)M}, c_n^{(\pm)}, \overline{c}_n^{(\pm)}\right)$ $(n\neq0)$ are as follows:

$$\left[\alpha_n^{(\pm)M}, \alpha_m^{(\pm)N}\right] = n\delta_{n+m,0}\eta^{MN}, \quad \left\{c_n^{(\pm)}, \overline{c}_m^{(\pm)}\right\} = \delta_{n+m,0}, \quad \text{others} = 0. \tag{A.7}$$

BRST charge $Q_{\rm B}$

The BRST charge in terms of the string coordinates and their conjugates is given (modulo normal ordering) by

$$Q_{\rm B} = 2\sqrt{\pi}\int_0^{2\pi}d\sigma\left\{i\pi_{\overline{c}}\left[-\frac{1}{2}\left(\eta^{MN}P_MP_N + \eta_{MN}X'^MX'^N\right) + i\left(c'\overline{c} - \pi_{\overline{c}}'\pi_c\right)\right]\right.$$

$$\left. -c\left(P_MX'^M + c'\pi_c + \pi_{\overline{c}}'\overline{c}\right)\right\}, \tag{A.8}$$

19

where the prime denotes the differentiation with respect to σ. In terms of the creation and the annihilation operators, Q_{B} in the $\pi_{\overline{c}}^0$-omitted formalism is

$$Q_{\mathrm{B}} = -\sum_{\pm}\sum_{n} : c_{-n}^{(\pm)}\left(\sum_{m}\left[\alpha_{n-m}^{(\pm)}\cdot\alpha_m^{(\pm)} + (n+m)\overline{c}_{n-m}^{(\pm)}c_m^{(\pm)}\right] - 2\delta_{n,0}\right): , \qquad (A.9)$$

with $\alpha_{0\,\mu}^{(\pm)} = p_\mu/2$ and $c_0 = \partial/\partial\overline{c}_0$. Making the dependence on the ghost zero-mode $\overline{c}_0$ explicit, Q_{B} is also expressed as

$$Q_{\mathrm{B}} = L\frac{\partial}{\partial\overline{c}_0} + \widetilde{Q}_{\mathrm{B}} + \overline{c}_0 M, \qquad (A.10)$$

where L and M are given by

$$L = -\frac{1}{2}p^2 - 2\sum_{\pm}\sum_{n\geq 1}\left[\alpha_{-n}^{(\pm)}\cdot\alpha_n^{(\pm)} + n\left(c_{-n}^{(\pm)}\overline{c}_n^{(\pm)} + \overline{c}_{-n}^{(\pm)}c_n^{(\pm)}\right)\right] + 4, \qquad (A.11)$$

$$M = 2\sum_{\pm}\sum_{n\geq 1} nc_{-n}^{(\pm)}c_n^{(\pm)}, \qquad (A.12)$$

and $\widetilde{Q}_{\mathrm{B}}$ is the part of Q_{B} (A.9) containing neither $\overline{c}_0$ nor c_0. $\widetilde{Q}_{\mathrm{B}}$ contains, in particular, the part

$$\widetilde{Q}_{\mathrm{B}} = i\sum_{\pm}\left(c_{-1}^{(\pm)}\alpha_1^{(\pm)\mu} + c_1^{(\pm)}\alpha_{-1}^{(\pm)\mu}\right)\frac{\partial}{\partial x^\mu} + \cdots , \qquad (A.13)$$

which contributes to $B(F)\cdot Q_{\mathrm{B}}\Lambda_\pm$.

Ghost number operator

The ghost number operator N_{gh} is given by

$$N_{\mathrm{gh}} = \overline{c}_0\frac{\partial}{\partial\overline{c}_0} + \widetilde{N}_{\mathrm{gh}}, \qquad (A.14)$$

with

$$\widetilde{N}_{\mathrm{gh}} = \sum_{\pm}\sum_{n\geq 1}\left(c_{-n}^{(\pm)}\overline{c}_n^{(\pm)} - \overline{c}_{-n}^{(\pm)}c_n^{(\pm)}\right). \qquad (A.15)$$

Dot and star products

For general string functionals Φ_i $(i = 1, 2, 3)$, the dot and the star products satisfy the following properties:

$$\Phi_1\cdot\Phi_2 = (-)^{|1||2|}\Phi_2\cdot\Phi_1, \qquad (A.16)$$

$$\Phi_1 * \Phi_2 = -(-)^{|1||2|}\Phi_2 * \Phi_1, \qquad (A.17)$$

$$\Phi_1\cdot(\Phi_2 * \Phi_3) = (-)^{|1|(|2|+|3|)}\Phi_2\cdot(\Phi_3 * \Phi_1), \qquad (A.18)$$

$$\Phi_1\cdot Q_{\mathrm{B}}\Phi_2 = -(-)^{|1|}(Q_{\mathrm{B}}\Phi_1)\cdot\Phi_2, \qquad (A.19)$$

where $|i|$ $(i = 1, 2, 3)$ is 0 (1) if Φ_i is Grassmann-even (-odd).

B   Derivation of $\Psi * \Lambda_\pm$

In this appendix, we present the details of the calculation of $\Psi * \Lambda_\pm$, eqs. (2.24) and (3.3). The following calculations are based on the (p_μ, α) representation, which is the Fourier transform of the $(x^\mu, \tilde{\alpha})$ representation adopted in the text. Therefore, the gauge transformation functional $\Lambda_\pm$ we have to consider here are given in bra form by

$$\langle \Lambda_\pm(p_\mu, \overline{c}_0, \alpha)| = i\overline{c}_0 \, \langle 0| \left(\alpha_{1\,\mu}^{(+)} \overline{c}_1^{(-)} \pm \alpha_{1\,\mu}^{(-)} \overline{c}_1^{(+)} \right) \varepsilon_\pm^\mu(p) \, \delta(\alpha) \times (2\pi)^{d+1}, \tag{B.1}$$

where $\varepsilon_+^\mu(p)$ and $\varepsilon_-^\mu(p)$ are the Fourier transforms of $\zeta^\mu(x)$ (2.23) and $\xi^\mu(x)$ (3.1), respectively:

$$\varepsilon_+^\mu(p) = ib^\mu{}_\nu \frac{\partial}{\partial p_\nu} \delta^d(p), \quad \varepsilon_-^\mu(p) = ia^\mu{}_\nu \frac{\partial}{\partial p_\nu} \delta^d(p). \tag{B.2}$$

The closed SFT three string vertex $|V(1,2,3)\rangle$ in the π_c^0-omitted formulation is

$$|V(1,2,3)\rangle = [\mu(1,2,3)]^2 \, \wp^{(1)} \wp^{(2)} \wp^{(3)}$$
$$\times \prod_{r=1}^3 \left(1 - \overline{c}_0^{(r)} \frac{1}{\sqrt{2}} w_I^{(r)} \right) \exp\left(F(1,2,3) \right) |0\rangle_{1,2,3} \, \delta(1,2,3), \tag{B.3}$$

with $F(1,2,3)$ and $\delta(1,2,3)$ given by

$$F(1,2,3) = \sum_\pm \sum_{r,s} \sum_{n,m \geq 1} \overline{N}_{nm}^{rs} \left(\frac{1}{2} \alpha_{-n}^{(\pm)(r)} \cdot \alpha_{-m}^{(\pm)(s)} + i\gamma_{-n}^{(\pm)(r)} \overline{\gamma}_{-m}^{(\pm)(s)} \right)$$
$$+ \frac{1}{2} \sum_\pm \sum_r \sum_{n \geq 1} \overline{N}_n^r \alpha_{-n}^{(\pm)(r)} \cdot \boldsymbol{P} + \tau_0 \sum_{r=1}^3 \frac{1}{\alpha_r} \frac{p_r^2}{4}, \tag{B.4}$$

$$\delta(1,2,3) = (2\pi)^d \delta^d \left(\sum_{r=1}^3 p_r \right) \cdot 2\pi \delta \left(\sum_{r=1}^3 \alpha_r \right). \tag{B.5}$$

Definitions of various quantities appearing in eqs. (B.3), (B.4) and (B.5) are found in [14, 30] (see also [27]). Some of them are given below when we use their explicit expressions. For later convenience we divide $F(1,2,3)$ (B.4) into two parts:

$$F = F_{\text{oscl}} + F_{p^2}, \qquad F_{p^2} \equiv \tau_0 \sum_{r=1}^3 \frac{1}{\alpha_r} \frac{p_r^2}{4}. \tag{B.6}$$

Now, what we have to calculate is

$$\int d1 \, \langle \Lambda_\pm(1)|V(1,2,3)\rangle = \wp^{(2)} \wp^{(3)} \lim_{\varepsilon \to 0} \lim_{p_1 \to 0} \begin{pmatrix} b_{\mu\nu} \\ a_{\mu\nu} \end{pmatrix} \frac{\partial}{\partial p_1^\nu} |A_\pm^\mu(1,2,3)\rangle_{2,3}, \tag{B.7}$$

with $|A_\pm^\mu(1,2,3)\rangle_{2,3}$ given by

$$|A_\pm^\mu(1,2,3)\rangle_{2,3} \equiv [\mu(1,2,3)]^2 \cdot (2\pi)^{d+1}\delta^d\Big(p_1 + \sum_{s=2,3} p_s\Big)\delta\Big(\varepsilon + \sum_{s=2,3}\alpha_s\Big)$$

$$\times {}_1\langle 0|\left(\alpha_1^{(+)\mu}\overline{c}_1^{(-)} \pm \alpha_1^{(-)\mu}\overline{c}_1^{(+)}\right)\prod_{r=2,3}\left(1 - \overline{c}_0^{(r)}\frac{1}{\sqrt{2}}w_I^{(r)}\right)\exp\left(F(1,2,3)\right)|0\rangle_{1,2,3} \ . \quad \text{(B.8)}$$

Here, we have used the abbreviation $d1 \equiv d\overline{c}_0^{(1)}d^dp_1/(2\pi)^d \cdot d\alpha_1/2\pi$, and ε is the string-length of the string 1, $\varepsilon \equiv \alpha_1$.

<u>Expansion of the quantities in V</u>

Eq. (B.7) tells that we have to take the $\varepsilon \to 0$ limit of the part of $|A_\pm^\mu(1,2,3)\rangle_{2,3}$ proportional to p_1^ν. For this purpose, we Laurent-expand various quantities in the vertex $V(1,2,3)$ with respect to ε/α_2. First, we have (c.f. Appendix A of ref. [27])

$$\tau_0 \equiv \sum_r \alpha_r \ln|\alpha_r| = \varepsilon\left\{\ln\left|\frac{\varepsilon}{e\alpha_2}\right| - \frac{\varepsilon}{2\alpha_2} + O(\varepsilon^2)\right\}, \quad \text{(B.9)}$$

$$[\mu(1,2,3)]^2 \equiv \exp\left(-2\tau_0\sum_{r=1}^{3}\frac{1}{\alpha_r}\right) = \left|\frac{e\alpha_2}{\varepsilon}\right|^2\left(1 + \frac{\varepsilon}{\alpha_2} + O(\varepsilon^2)\right), \quad \text{(B.10)}$$

$$\sum_{r=1}^{3}\frac{1}{\alpha_r}\frac{p_r^2}{4} = \frac{1}{4\varepsilon}\left(p_1^2 - p_1\cdot(p_1 + 2p_2)\frac{\varepsilon}{\alpha_2} + O(\varepsilon^2)\right), \quad \text{(B.11)}$$

where $O(\varepsilon)$ represents precisely $O(\varepsilon/\alpha_2)$. Therefore, F_{p^2} of (B.6) is expanded as

$$F_{p^2} = -\frac{1}{4}(p_1)^2\ln\left|\frac{e\alpha_2}{\varepsilon}\right| + O\left(\varepsilon p_1\ln\left|\frac{e\alpha_2}{\varepsilon}\right|\right) + O\left(\varepsilon(p_1)^2\right) + O(\varepsilon^2). \quad \text{(B.12)}$$

Next, let us expand the quantities associated with the creation/annihilation operators. For the Neumann coefficients, we have

$$\overline{N}_1^1 = \frac{1}{\varepsilon}e^{\tau_0/\varepsilon} = \frac{\text{sgn}(\varepsilon\alpha_2)}{e\alpha_2}\left(1 - \frac{\varepsilon}{2\alpha_2} + O(\varepsilon^2)\right), \quad \text{(B.13)}$$

$$\overline{N}_m^s = \frac{1}{m\alpha_2}\times\begin{cases}1 & (s=2)\\(-)^m & (s=3)\end{cases} + \cdots, \quad \text{(B.14)}$$

$$\overline{N}_{1m}^{1s} = \left|\frac{\varepsilon}{e\alpha_2}\right|\times\begin{cases}1 & (s=2)\\-(-)^m & (s=3)\end{cases} + \cdots, \quad \text{(B.15)}$$

$$\overline{N}_{nm}^{23} = -\frac{(-)^n}{n}\delta_{n,m}\left(1 + O(\varepsilon^2)\right) - (1 - \delta_{n,m})\frac{(-)^n}{n-m}\frac{\varepsilon}{\alpha_2} + \cdots, \quad \text{(B.16)}$$

where the dots $\ldots$ represents terms of higher order in ε/α_2. Using (B.14) and (B.16), F_{oscl} with the creation operators of the string 1 set equal to zero is expanded as follows:

$$F_{\text{oscl}}(1,2,3)\Big|_{\text{oscl}^{(1)}\equiv 0} = -\sum_{\pm}\sum_{n\geq 1}(-)^n\left(\frac{1}{n}\alpha_{-n}^{(\pm)(2)}\cdot\alpha_{-n}^{(\pm)(3)} - c_{-n}^{(\pm)(2)}\overline{c}_{-n}^{(\pm)(3)} + \overline{c}_{-n}^{(\pm)(2)}c_{-n}^{(\pm)(3)}\right)$$

$$
-\frac{1}{2}\sum_{\pm}\sum_{n\geq1}\frac{1}{n}\left(\alpha_{-n}^{(\pm)(2)}+(-)^n\alpha_{-n}^{(\pm)(3)}\right)\cdot p_1-\frac{\varepsilon}{\alpha_2}\sum_{\pm}\sum_{n\geq1}(-)^n\left(c_{-n}^{(\pm)(2)}\overline{c}_{-n}^{(\pm)(3)}-c_{-n}^{(\pm)(3)}\overline{c}_{-n}^{(\pm)(2)}\right)
$$

$$
+\frac{\varepsilon}{\alpha_2}\times\left(\alpha_{-n}^{(2)}\cdot\alpha_{-m}^{(3)}\text{ and }c_{-n}^{(2,3)}\overline{c}_{-m}^{(3,2)}\text{ with }n\neq m\right)+O(\varepsilon^2). \tag{B.17}
$$

The fourth term on the RHS of (B.17) does not contribute to our final result due to the projector $\wp$ and hence will be omitted hereafter. Then, from (B.17) and (B.12), we obtain

$$
[\mu(1,2,3)]^2\exp\left(F_{\mathrm{oscl}}\big|_{\mathrm{oscl}^{(1)}\equiv0}+F_{p^2}\right)(2\pi)^{d+1}\delta\Big(p_1+\sum_{s=2,3}p_s\Big)\delta\Big(\varepsilon+\sum_{s=2,3}\alpha_s\Big)
$$

$$
=\left|\frac{e\alpha_2}{\varepsilon}\right|^2\left\{1+\frac{\varepsilon}{\alpha_2}\left(1-\widetilde{N}_{\mathrm{gh}}^{(2)}\right)\right\}
$$

$$
\times:\exp\left\{-ip_1^\mu\sqrt{\pi}X^{(2)\mu}(\sigma=0)\right\}:\left(1+\varepsilon\frac{\partial}{\partial\alpha_2}\right)|r(2,3)\rangle, \tag{B.18}
$$

where $\widetilde{N}_{\mathrm{gh}}$ is given by (A.15), and $|r(2,3)\rangle$ is the reflector (without the $\overline{c}_0$-part),

$$
|r(2,3)\rangle\equiv\exp\left\{-\sum_{\pm}\sum_{n\geq1}(-)^n\left(\frac{1}{n}\alpha_{-n}^{(\pm)(2)}\cdot\alpha_{-n}^{(\pm)(3)}-c_{-n}^{(\pm)(2)}\overline{c}_{-n}^{(\pm)(3)}+\overline{c}_{-n}^{(\pm)(2)}c_{-n}^{(\pm)(3)}\right)\right\}|0\rangle_{2,3}
$$

$$
\times(2\pi)^d\delta^d(p_2+p_3)\cdot2\pi\delta(\alpha_2+\alpha_3), \tag{B.19}
$$

which enjoys the following property,

$$
\begin{pmatrix}\alpha_n^{(\pm)(2)}+(-)^n\alpha_{-n}^{(\pm)(3)}\\c_n^{(\pm)(2)}+(-)^nc_{-n}^{(\pm)(3)}\\\overline{c}_n^{(\pm)(2)}-(-)^n\overline{c}_{-n}^{(\pm)(3)}\end{pmatrix}|r(2,3)\rangle=0\quad(n=\pm1,\pm2,\dots). \tag{B.20}
$$

Two comments are in order for eq. (B.18). First, as seen from eq. (B.12), F_{p^2} does not contribute to (B.7) and hence is omitted on the RHS of (B.18). However, when we consider $\zeta^\mu(x)$ and $\xi^\mu(x)$ of higher power than linear in x^μ, we have to take F_{p^2} into account. Second, $\sqrt{\pi}X^{(2)\mu}(\sigma=0)$ in (B.18) originally appears as

$$
i\left(\partial/\partial p_2^\mu\right)-(i/2)\sum_{\pm}\sum_{n\geq1}\frac{1}{n}\left(\alpha_{-n}^{(\pm)\mu(2)}+(-)^n\alpha_{-n}^{(\pm)\mu(3)}\right), \tag{B.21}
$$

consisting solely of the creation operators. Therefore, we need the normal ordering symbol in eq. (B.18).

<u>Formulas for contractions</u>

Our next task is to obtain the expressions of various "contractions" which appear in the calculation of $_1\langle0|\cdots|0\rangle_{1,2,3}$ in (B.8). First, we have

$$
\overline{\alpha_1^{(\pm)\mu(1)}\,F}=\sum_{s=2,3}\sum_{m\geq1}\overline{N}_{1m}^{1s}\alpha_{-m}^{(\pm)\mu(s)}+\frac{1}{2}\overline{N}_1^1\boldsymbol{P}^\mu
$$

23

$$= \left| \frac{\varepsilon}{e\alpha_2} \right| \left\{ \sum_{n \geq 1} \left(\alpha_{-n}^{(\pm)\mu(2)} - (-)^n \alpha_{-n}^{(\pm)\mu(3)} \right) + \frac{1}{2} p_2^\mu - \left(\frac{\alpha_2}{2\varepsilon} - \frac{1}{4} \right) p_1^\mu + O(\varepsilon) \right\},$$

$$\text{(B.22)}$$

$$\overline{c}_1^{(\mp)(1)} F = \sum_{s=2,3} \sum_{m \geq 1} \overline{N}_{1m}^{1s} \frac{\varepsilon}{\alpha_s} \overline{c}_{-m}^{(\mp)(s)}$$

$$= \sum_{m \geq 1} \left| \frac{\varepsilon}{e\alpha_2} \right| \frac{\varepsilon}{\alpha_2} \left(\overline{c}_{-m}^{(\mp)(2)} + (-)^m \overline{c}_{-m}^{(\mp)(3)} \right) + O(\varepsilon^3). \tag{B.23}$$

Using the expansion (c.f., eq. (A.6) of [27])

$$w_{1,1}^{(r)} = (-)^{r-1} \frac{1}{\varepsilon} \left| \frac{\varepsilon}{e\alpha_2} \right| \left(1 - \frac{\varepsilon}{2\alpha_2} + O(\varepsilon^2) \right), \tag{B.24}$$

for the coefficient $w_{1,1}^{(r)}$ $(r = 2, 3)$ in

$$w_I^{(r)} \equiv \frac{i}{\sqrt{2}} \sum_{\pm} \sum_{s=1}^{3} \sum_{n \geq 1} w_{n,s}^{(r)} \gamma_{-n}^{(\pm)(s)}, \tag{B.25}$$

we get

$$\overline{c}_1^{(\mp)(1)} w_I^{(r)} = \overline{c}_1^{(\mp)(1)} \frac{i}{\sqrt{2}} w_{1,1}^{(r)} \gamma_{-1}^{(\mp)(1)} = \frac{1}{\sqrt{2}} \varepsilon w_{1,1}^{(r)}$$

$$= (-)^{r-1} \frac{1}{\sqrt{2}} \left| \frac{\varepsilon}{e\alpha_2} \right| \left(1 - \frac{\varepsilon}{2\alpha_2} + O(\varepsilon^2) \right) \quad (r = 2, 3). \tag{B.26}$$

From

$$w_{n,s}^{(r)} = (-)^r \frac{1}{n\alpha_2} \times \begin{cases} 1 & (s = 2) \\ (-)^n & (s = 3) \end{cases} + O(\varepsilon) \quad (r = 2, 3), \tag{B.27}$$

which is a special case of eq. (A.6) of [27], we have for $w_I^{(r=2,3)}$ left after the contractions:

$$\widetilde{w}_I^{(r=2,3)} \equiv w_I^{(r)} \Big|_{c^{(1)}=0} = \frac{i}{\sqrt{2}} \sum_{\pm} \sum_{s=2,3} \sum_{n \geq 1} w_{n,s}^{(r)} \gamma_{-n}^{(\pm)(s)}$$

$$= (-)^r \frac{1}{\sqrt{2}} \sum_{\pm} \sum_{n \geq 1} \left(c_{-n}^{(\pm)(2)} - (-)^n c_{-n}^{(\pm)(3)} \right) + O(\varepsilon). \tag{B.28}$$

Contractions I : $\overline{\alpha_1^{(\pm)\mu(1)} F \cdot \overline{c}_1^{(\mp)(1)} w_I^{(r=2,3)}}$

Let us consider all possible contractions which appear in $_1\langle 0| \cdots |0\rangle_{1,2,3}$ of (B.8). First is the contraction of the type $\overline{\alpha_1^{(\pm)\mu(1)} F \cdot \overline{c}_1^{(\mp)(1)} w_I^{(r=2,3)}}$. It has contributions from the second and the last terms on the RHS of

$$\prod_{r=2,3} \left(1 - \overline{c}_0^{(r)} \frac{1}{\sqrt{2}} w_I^{(r)} \right) = 1 + \sum_{r=2,3} \left(-\overline{c}_0^{(r)} \frac{1}{\sqrt{2}} w_I^{(r)} \right) + \overline{c}_0^{(2)} \frac{1}{\sqrt{2}} w_I^{(2)} \overline{c}_0^{(3)} \frac{1}{\sqrt{2}} w_I^{(3)}. \tag{B.29}$$

24

The contribution from the second term is

$$\overbrace{\alpha_1^{(\pm)\mu(1)}\overbrace{\bar{c}^{(\mp)(1)}\sum_{r=2,3}\left(-\bar{c}_0^{(r)}\frac{1}{\sqrt{2}}w_I^{(r)}\right)}F} = -\left(\bar{c}_0^{(2)}-\bar{c}_0^{(3)}\right)\frac{1}{2}\left(\frac{\varepsilon}{e\alpha_2}\right)^2\left(1-\frac{\varepsilon}{2\alpha_2}\right)$$

$$\times\left\{\sum_{n\geq 1}\left[\alpha_{-n}^{(\pm)\mu(2)}-(-)^n\alpha_{-n}^{(\pm)\mu(3)}\right]+\frac{1}{2}p_2^\mu-\left(\frac{\alpha_2}{2\varepsilon}-\frac{1}{4}\right)p_1^\mu\right\}. \tag{B.30}$$

Corresponding to $\langle\Lambda_-|$ ($\langle\Lambda_+|$), we anti-symmetrize (symmetrize) (B.30) with respect to ($\pm$):

$$\text{Anti-symm.: }\sum_\pm(\pm)(\text{B.30})=\sqrt{\pi}\left(\bar{c}_0^{(2)}-\bar{c}_0^{(3)}\right)\left(\frac{\varepsilon}{e\alpha_2}\right)^2\frac{d}{d\sigma}X^{\mu(2)}(\sigma)\bigg|_{\sigma=0}, \tag{B.31}$$

$$\text{Symm.: }\sum_\pm(\text{B.30})=-\frac{1}{2}\left(\bar{c}_0^{(2)}-\bar{c}_0^{(3)}\right)\left(\frac{\varepsilon}{e\alpha_2}\right)^2\left(1-\frac{\varepsilon}{2\alpha_2}\right)$$

$$\times\left\{2\sqrt{\pi}P_\mu^{(2)}(\sigma=0)-\left(\frac{\alpha_2}{\varepsilon}-\frac{1}{2}\right)p_1^\mu\right\}. \tag{B.32}$$

In eqs. (B.31) and (B.32), we have converted the creation operators of the string 3 into the annihilation ones of the string 2 using (B.20) and hence we have $(d/d\sigma)X^{\mu(2)}$ and $P_\mu^{(2)}$ there.

Next, as the contribution from the last term of (B.29), we obtain

$$\overbrace{\alpha_1^{(\pm)\mu(1)}F}\left(\overbrace{\bar{c}^{(\mp)(1)}\frac{1}{\sqrt{2}}\bar{c}_0^{(2)}w_I^{(2)}}\frac{1}{\sqrt{2}}\bar{c}_0^{(3)}\tilde{w}_I^{(3)}+\overbrace{\bar{c}^{(\mp)(1)}\frac{1}{\sqrt{2}}\bar{c}_0^{(2)}\tilde{w}_I^{(2)}}\frac{1}{\sqrt{2}}\bar{c}_0^{(3)}w_I^{(3)}\right)$$

$$=-\frac{1}{2}\overbrace{\alpha_1^{(\pm)\mu(1)}F}\,\overbrace{\bar{c}^{(\pm)(1)}w_I^{(2)}}\left(\tilde{w}_I^{(3)}+\tilde{w}_I^{(2)}\right)\bar{c}_0^{(2)}\bar{c}_0^{(3)}+\dots$$

$$=O(\varepsilon^2)\times p_1^\mu\times\left(\text{terms linear in }c_{-n}^{(2,3)}\right), \tag{B.33}$$

where we have used the fact that $\tilde{w}_I^{(3)}+\tilde{w}_I^{(2)}=O(\varepsilon)$ as seen from (B.28). Eq. (B.33) is manifestly symmetric with respect to ($\pm$).

<u>Contractions II : $\overbrace{\alpha_1^{(\pm)\mu(1)}F\cdot\bar{c}_1^{(\mp)(1)}F}$</u>

Since $\tilde{w}_I^{(2)}\tilde{w}_I^{(3)}=O(\varepsilon)$, we have

$$\prod_{r=2,3}\left(1-\bar{c}_0^{(r)}\frac{1}{\sqrt{2}}\tilde{w}_I^{(r)}\right)=1+\sum_{r=2,3}\left(-\bar{c}_0^{(r)}\frac{1}{\sqrt{2}}\tilde{w}_I^{(r)}\right)-\frac{1}{2}\bar{c}_0^{(2)}\bar{c}_0^{(3)}\tilde{w}_I^{(2)}\tilde{w}_I^{(3)}$$

$$=1-\frac{1}{\sqrt{2}}\left(\bar{c}_0^{(2)}-\bar{c}_0^{(3)}\right)\tilde{w}_I^{(2)}+O(\varepsilon), \tag{B.34}$$

and hence

$$\overbrace{\alpha_1^{(\pm)\mu(1)}\overbrace{\bar{c}^{(\mp)(1)}\prod_{r=2,3}\left(1-\bar{c}_0^{(r)}\frac{1}{\sqrt{2}}\tilde{w}_I^{(r)}\right)F}F}$$

25

$$= O(\varepsilon^2) \times p_1^\mu \times \left(\text{terms linear in } c_{-n}^{(2,3)} \right)$$

$$- \frac{1}{4} p_1^\mu \left| \frac{\varepsilon}{e\alpha_2} \right|^2 \left(\overline{c}_0^{(2)} - \overline{c}_0^{(3)} \right) \sum_{m \geq 1} \left(\overline{c}_{-m}^{(\mp)(2)} + (-)^m \overline{c}_{-m}^{(\mp)(3)} \right) \sum_{\pm} \sum_{n \geq 1} \left(c_{-n}^{(\pm)(2)} - (-)^n c_{-n}^{(\pm)(3)} \right).$$

$$(B.35)$$

Among many terms on the RHS of (B.22), only the $O(\varepsilon^0)$ term, $|\varepsilon/e\alpha_2| \, (-\alpha_2/2\varepsilon) \, p_1^\mu$, contributes to (B.35). (Anti-)symmetrizing (B.35) with respect to $(\pm)$, we obtain

$$\text{Anti-symm.:} \quad \sum_\pm (\pm)(\text{B.35}) = O(\varepsilon^2) \times p_1^\mu \times \left(\text{terms linear in } c_{-n}^{(2,3)} \right)$$

$$- \pi p_1^\mu \left| \frac{\varepsilon}{e\alpha_2} \right|^2 \left(\overline{c}_0^{(2)} - \overline{c}_0^{(3)} \right) : i\pi_c^{(2)}|_{\text{oscl}}(\sigma = 0) \cdot i\pi_{\overline{c}}^{(2)}|_{\text{oscl}}(\sigma = 0) : , \quad (B.36)$$

$$\text{Symm.:} \quad \sum_\pm (\text{B.35}) = O(\varepsilon^2) \times p_1^\mu \times \left(\text{terms linear in } c_{-n}^{(2,3)} \right)$$

$$- \pi p_1^\mu \left| \frac{\varepsilon}{e\alpha_2} \right|^2 \left(\overline{c}_0^{(2)} - \overline{c}_0^{(3)} \right) : \overline{c}|_{\text{oscl}}(\sigma = 0) \cdot i\pi_{\overline{c}}|_{\text{oscl}}(\sigma = 0) : , \quad (B.37)$$

where "oscl" indicates the creation/annihilation operator parts.

<u>Evaluation of (B.7)</u>

Having obtained all the necessary formulas, let us evaluate (B.7). In the following we use the abbreviation $\wp^{(2,3)} \equiv \wp^{(2)} \wp^{(3)}$. First is (B.7) for Λ_-:

$$\int d1 \, \langle \Lambda_-(1)|V(1,2,3)\rangle = a_{\mu\nu} \wp^{(2,3)} \lim_{\varepsilon \to 0} \lim_{p_1 \to 0} \frac{\partial}{\partial p_1^\nu} \Big((\text{B.31}) + (\text{B.36}) \Big) \times (\text{B.18})$$

$$= a_{\mu\nu} \wp^{(2,3)} \left(-i\pi : \frac{dX^\mu}{d\sigma} \cdot X^\nu : -\pi\eta^{\mu\nu} : i\pi_c|_{\text{oscl}} \, i\pi_{\overline{c}}|_{\text{oscl}} : \right)^{(2)} \Bigg|_{\sigma=0} |R(2,3)\rangle$$

$$= \frac{1}{2} a_{\mu\nu} \left(-i \int_0^{2\pi} d\sigma \frac{dX^\mu(\sigma)}{d\sigma} X^\nu(\sigma) - \eta^{\mu\nu} \int_0^{2\pi} d\sigma \, i\pi_c(\sigma)|_{\text{oscl}} \cdot i\pi_{\overline{c}}(\sigma)|_{\text{oscl}} \right)^{(2)} |R(2,3)\rangle,$$

$$(B.38)$$

where $|R(2,3)\rangle$ is the full reflector,

$$|R(2,3)\rangle \equiv |r(2,3)\rangle \times \left(\overline{c}_0^{(2)} - \overline{c}_0^{(3)} \right). \quad (B.39)$$

In obtaining (B.38) we have used the fact that the first term on the RHS of (B.36) does not contribute due to the presence of $\wp^{(2,3)}$.

Next, (B.7) for Λ_+ is

$$\int d1 \, \langle \Lambda_+(1)|V(1,2,3)\rangle = b_{\mu\nu} \wp^{(2,3)} \lim_{\varepsilon \to 0} \lim_{p_1 \to 0} \frac{\partial}{\partial p_1^\nu} \Big((\text{B.32}) + (\text{B.33}) + (\text{B.37}) \Big) \times (\text{B.18})$$

$$= b_{\mu\nu} \wp^{(2,3)} \left\{ i\pi : P_\mu^{(2)} X_\nu^{(2)} : -\pi\eta^{\mu\nu} : \overline{c}^{(2)}|_{\mathrm{oscl}}\, i\pi_{\overline{c}}^{(2)}|_{\mathrm{oscl}} : + \frac{1}{2}\eta^{\mu\nu} \left(\alpha_2 \frac{\partial}{\partial\alpha_2} - \widetilde{N}_{\mathrm{gh}}^{(2)} \right) \right\}_{\sigma=0} |R(2,3)\rangle$$

$$= \frac{1}{2} b_{\mu\nu} \left(\frac{i}{2} \int_0^{2\pi} d\sigma \left\{ P_\mu(\sigma), X_\nu(\sigma) \right\} + \frac{1}{2}\eta^{\mu\nu} \left\{ \alpha, \frac{\partial}{\partial\alpha} \right\} - \eta_{\mu\nu}\mathcal{G} \right)^{(2)} |R(2,3)\rangle . \tag{B.40}$$

The points in deriving (B.40) are as follows:

- Due to the $-(\alpha_2/\varepsilon)p_1^\mu$ term in (B.32), we have to take into account the next-to-leading terms in (B.18) and (B.32). In particular, $\varepsilon\alpha_2\,(\partial/\partial\alpha_2)$ in (B.18) does contribute.

- $\delta(\alpha)$ in $\langle\Lambda_\pm|$ (5.16) should be understood to imply

$$\delta(\alpha) \equiv \lim_{\varepsilon\to+0} \frac{1}{2}\Big(\delta(\alpha-\varepsilon) + \delta(\alpha+\varepsilon)\Big), \tag{B.41}$$

  owing to the hermiticity of $\Lambda_\pm$. Therefore, the $O(1/\varepsilon)$ term originating from the term $-(\alpha_2/\varepsilon)p_1^\mu$ in (B.32) is missing from (B.40).

- Neither the first term on the RHS of (B.37) nor (B.33) contribute owing to the presence of $\wp^{(2,3)}$.

Eqs. (2.24) and (3.3) are consequences of eqs. (B.38) and (B.40) as well as the formula for the star product,

$$|(\Psi * \Lambda)(2)\rangle = \epsilon_\Psi \epsilon_\Lambda \int d3 \int d1 \, \langle\Psi(3)| \, \langle\Lambda(1)||V(1,2,3)\rangle , \tag{B.42}$$

where the sign factor ϵ_Ψ is 1 (-1) if Ψ is hermitian (anti-hermitian):

$$\langle\Psi(2)| = \epsilon_\Psi \int d1 \, \langle R(1,2)|\Psi(1)\rangle . \tag{B.43}$$

The transformation functional Λ is anti-hermitian and we have $\epsilon_\Lambda = -1$. Note that the integration measure $\int d1$ is Grassmann-odd and anti-hermitian, $(\int d1)^\dagger = -\int d1$, and hence eq. (B.43) implies $|\Psi(2)\rangle = \epsilon_\Psi(-)^{|\Psi|} \int d1 \, \langle\Psi(1)|R(1,2)\rangle$, where $|\Psi| = 0$ (1) if Ψ is Grassmann-even (-odd).

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